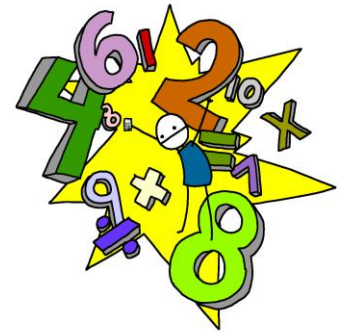
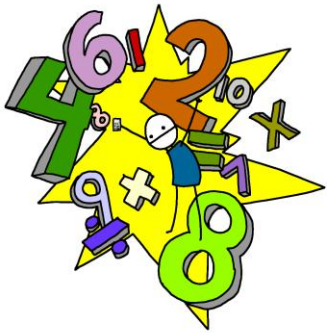


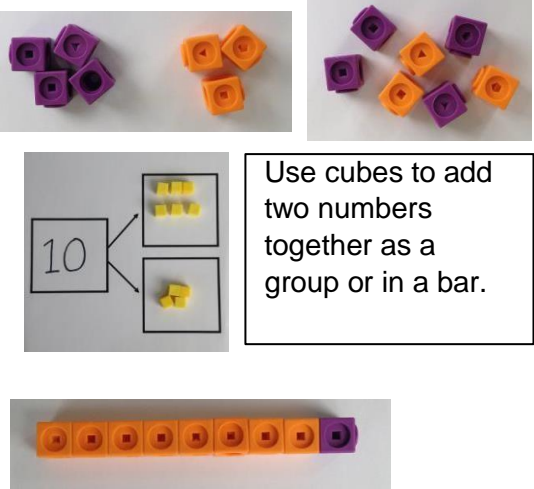
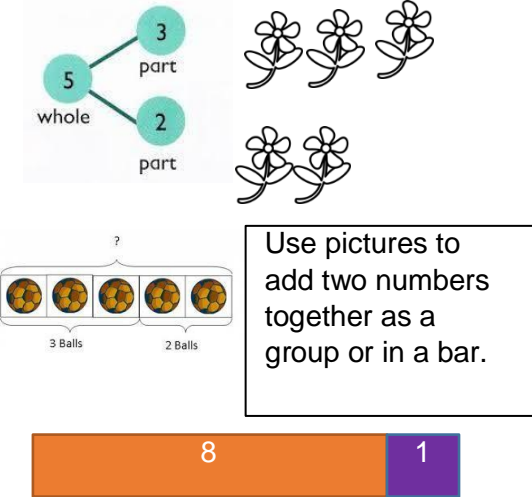
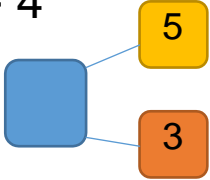

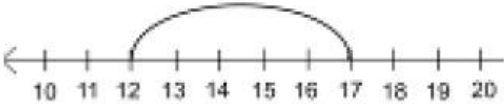


Bugle School Calculation Policy January 2018



Progression in Calculations

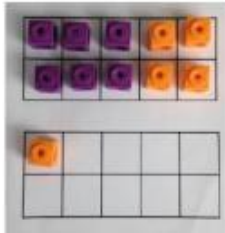
Addition

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model</p>	 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number on a bead string and then count on to the smaller number 1 by 1 to find the answer. This could also be done with cubes and counters.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>

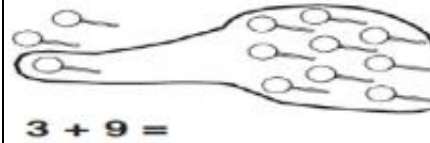
Regrouping to make 10.



$$6 + 5 = 11$$

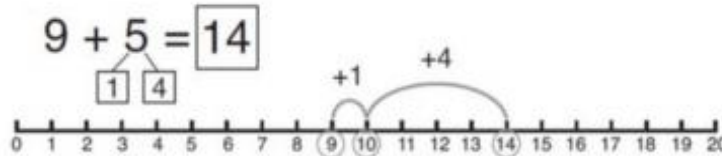


Start with the bigger number and use the smaller number to make 10.



$$3 + 9 =$$

Use pictures or a number line. Regroup or partition the smaller number to make 10.



$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10. How many more do I add on now?

Adding three single digits

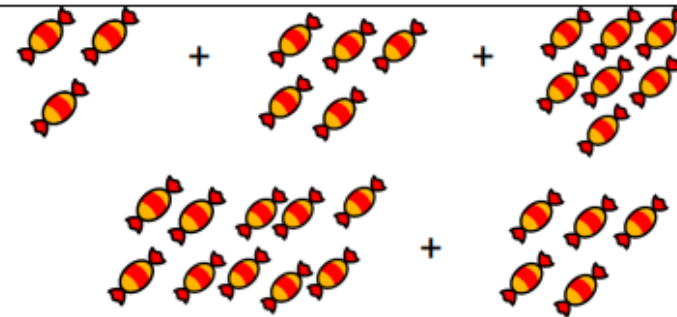
$$4 + 7 + 6 = 17$$

Put 4 and 6 together to make 10. Add on 7.



Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.

Following on from making 10, make 10



Add together three groups of objects. Draw a picture to recombine the groups to make 10.

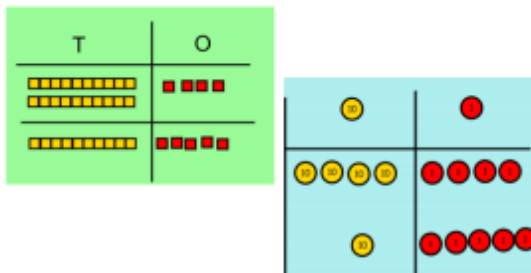
$$\begin{array}{l} \textcircled{4} + 7 + \textcircled{6} = \boxed{10} + \boxed{7} \\ \quad \quad \quad 10 \\ \quad \quad \quad = \boxed{17} \end{array}$$

Combine the two numbers that make 10 and then add on the remainder.

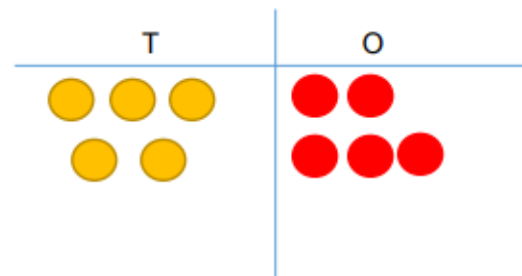
Column method- no regrouping

$$24 + 15 =$$

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.



After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.



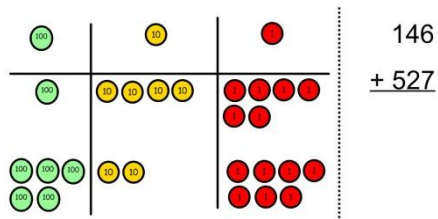
Calculations

$$21 + 42 =$$

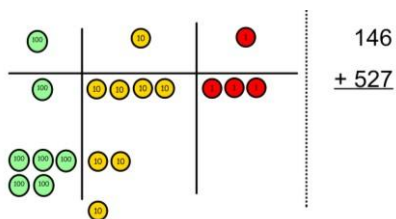
$$\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$$

Column method-regrouping

Make both numbers on a place value Grid (use base 10 first before moving on to place value counters).



Add up the units and exchange 10 ones for one 10.

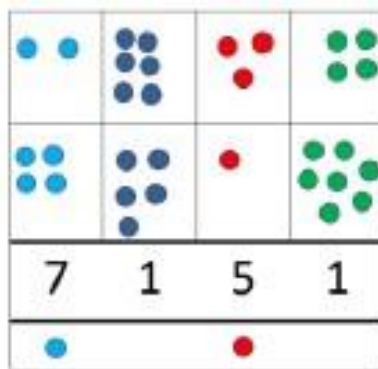


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

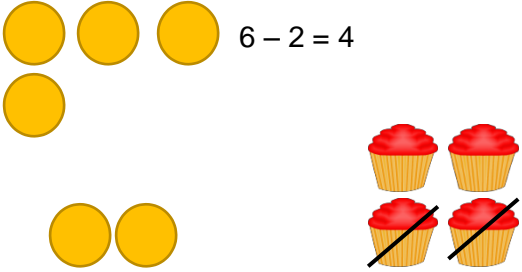
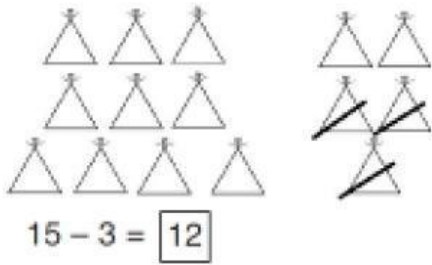


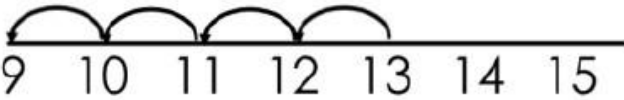
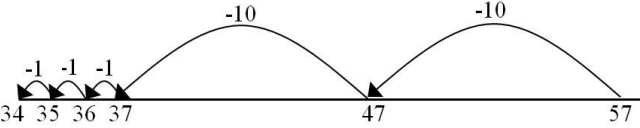
$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$$

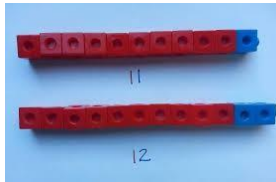
$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Subtraction

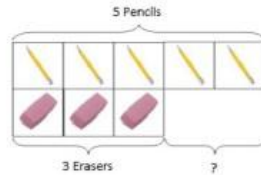
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 2 = 4$</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p>	<p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p>
<p>Counting back</p>	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>$13 - 4$</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>

Find the difference

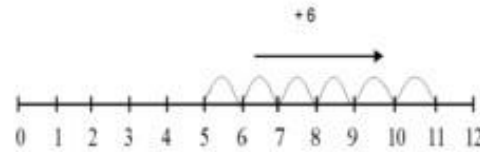
Compare amounts and objects to find the difference.



Use cubes to build towers or make bars to find the difference



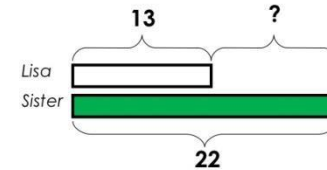
Use basic bar models with items to find the difference



Count on to find the difference.

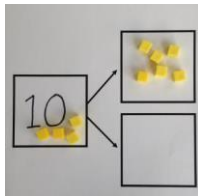
Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old.
Find the difference in age between them.



Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Part Part Whole Model

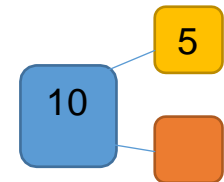
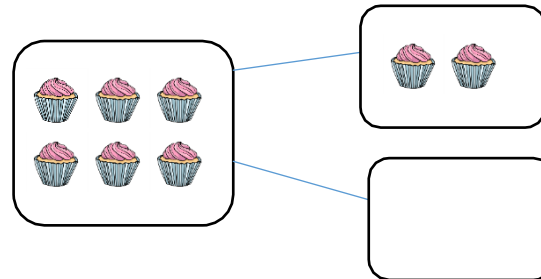


Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

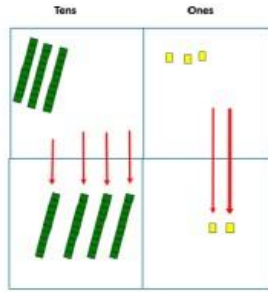
$$10 - 6 =$$

Use a pictorial representation of objects to show the part part whole model.



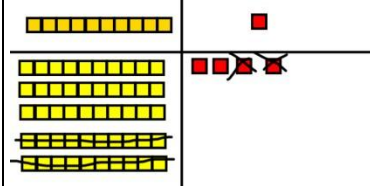
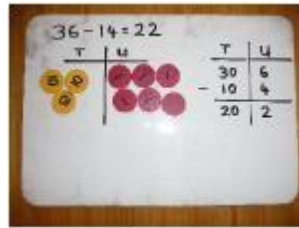
Move to using numbers within the part whole model.

Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to help to show working.

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

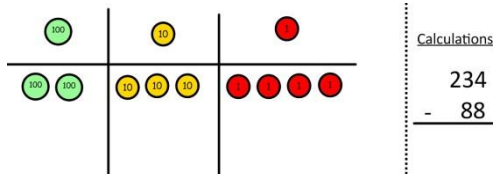
This will lead to a clear written column subtraction.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

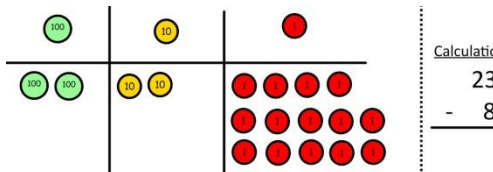
Make the larger number with the place value counters



Calculations

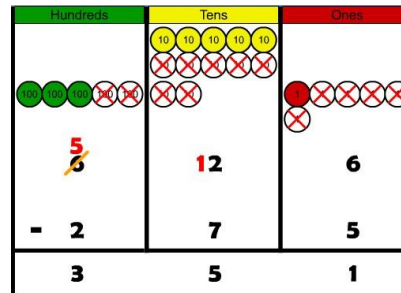
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

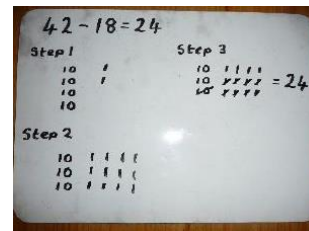


Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

$$836 - 254 = 582$$

$$\begin{array}{r} 800 + 30 + 6 \\ - 200 + 50 + 4 \\ \hline 500 + 80 + 2 \end{array}$$

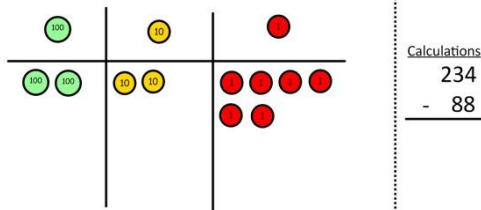
Children can start their formal written method by partitioning the number into clear place value columns.

$$728 - 582 = 146$$

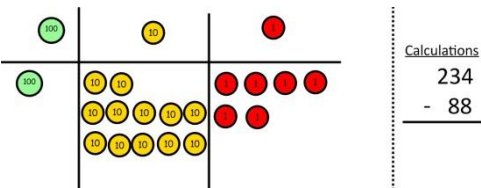
$$\begin{array}{r} 700 + 20 + 8 \\ - 500 + 80 + 2 \\ \hline 100 + 40 + 6 \end{array}$$

Moving forward the children use a more compact method.

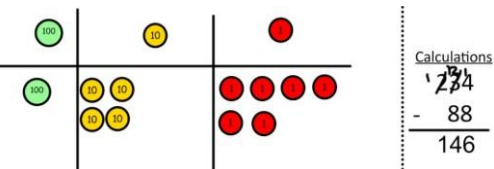
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction

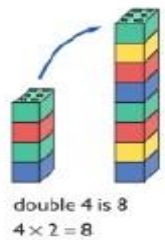

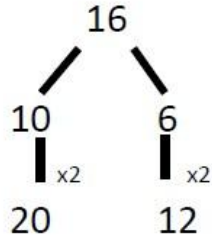




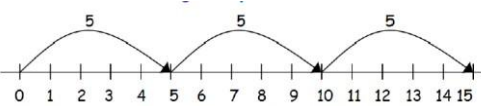



Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad \cancel{0} \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

Multiplication

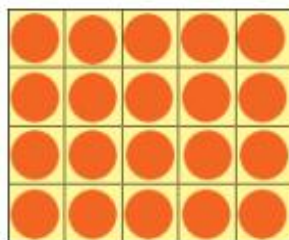
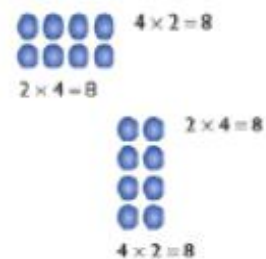
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities to show how to double a number.</p> 	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
Repeated addition	   <p>Use different objects to add equal groups.</p>	<p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>   <p>$5 + 5 + 5 = 15$</p>	<p>Write addition sentences to describe objects and pictures.</p>  <p>$2 + 2 + 2 + 2 + 2 = 10$</p>

Arrays- showing commutative multiplication

Create arrays using counters/ cubes to show multiplication sentences.



Draw arrays in different rotations to find **commutative** multiplication sentences.



Link arrays to area of rectangles.

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

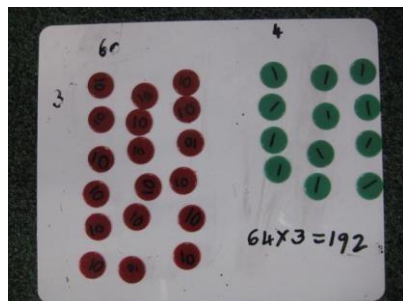
$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

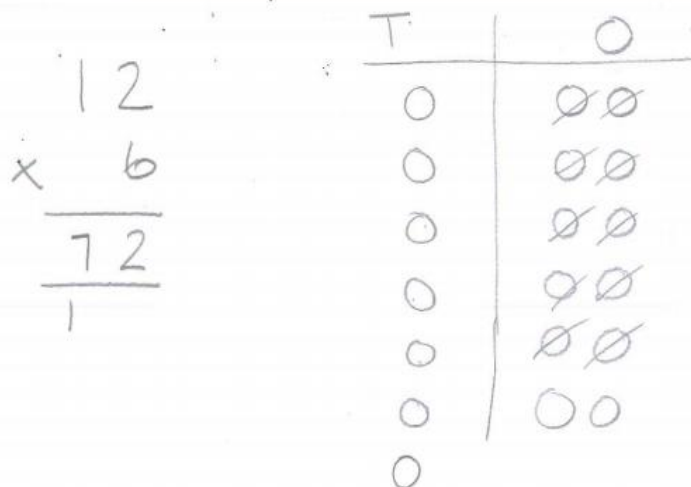
Column multiplication

Begin by using base 10 then place value counters in an array fashion with ones and tens laid out in a place value grid.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Children can then use pictorial representations of base 10 or place value counters to support their understanding (including exchanging) alongside the written column method.



Start with expanded column multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

$$\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$$

$$\begin{array}{r} 74 \\ \times 63 \\ \hline 12 \\ 210 \\ 240 \\ + 4200 \\ \hline 4662 \end{array}$$

This moves to the more compact method.



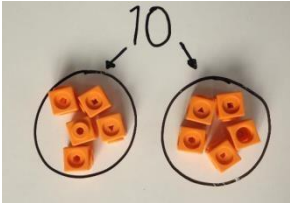
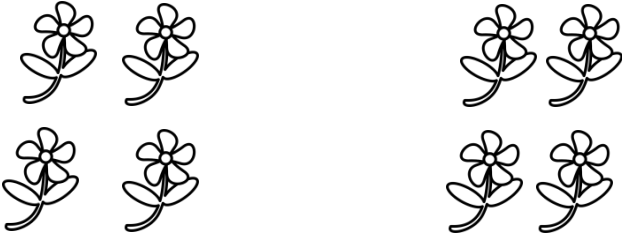
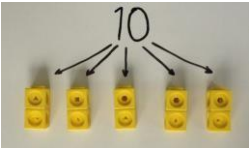
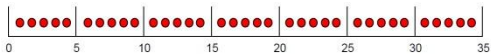
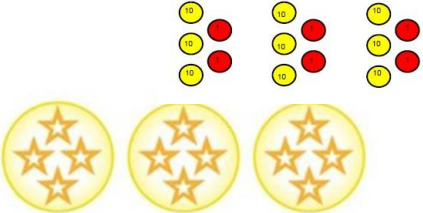
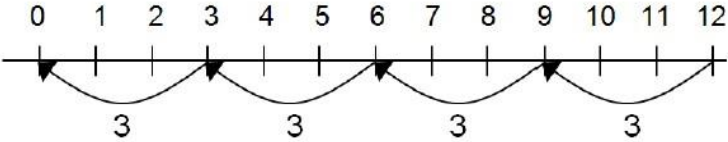

$$7 \times 22 = 154$$

$$\begin{array}{r} 7 \\ \times 22 \\ \hline 154 \\ \hline \end{array}$$

$$51 \times 26 = 1376$$

$$\begin{array}{r} 51 \\ \times 26 \\ \hline 356 \\ + 1020 \\ \hline 1376 \end{array}$$

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	<div></div> <div></div> <div></div> <div>I have 10 cubes, can you share them equally in 2 groups?</div>	<div>Children use pictures or shapes to share quantities.</div> <div></div> <div><div>8 ÷ 2 = 4</div></div>	<div>Share 9 buns between three people.</div> <div><div>9 ÷ 3 = 3</div></div>
Division as grouping	<div>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</div> <div></div> <div></div> <div><div>96 ÷ 3 = 32</div></div> <div></div>	<div>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</div> <div></div> <div>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</div> <div></div> <div><div>20 ÷ 5 = ?</div><div>5 x ? = 20</div></div>	<div>28 ÷ 7 = 4</div> <div>Divide 28 into 7 groups. How many are in each group?</div>

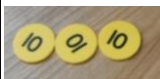
<h3>Division using arrays</h3>	<div data-bbox="405 132 725 339"> </div> <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<div data-bbox="969 132 1630 408"> </div> <p>Draw an array and use lines to split the array into groups to make multiplication and division sentences.</p>	<p>Find the inverse of multiplication and division sentences by creating four linking number sentences.</p> <p>$7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$</p>
<h3>Division using arrays with a remainder</h3>	<p>$14 \div 3 =$ Divide objects between groups and see how much is left over</p> <div data-bbox="409 707 931 1074"> </div>	<p>Draw dots and group them to divide an amount and clearly show a remainder.</p> <div data-bbox="1059 930 1514 1026"> </div>	<p>Complete written divisions and show the remainder using r.</p> <div data-bbox="1731 699 2067 770"> $\begin{array}{ccccccc} 29 & \div & 8 & = & 3 & \text{REMAINDER} & 5 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{dividend} & & \text{divisor} & & \text{quotient} & & \text{remainder} \end{array}$ </div>

Division on a number line (with or without remainders)

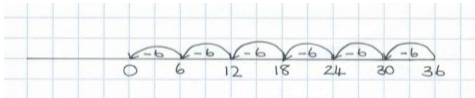
1) Make the 2 digit number out of the place value counters and draw a number line, recording the largest number at the right hand end of the number line.



2) Subtract the smallest number from the starting number by removing the appropriate number of counters. Record this by making a jump backwards on the number line (putting within the jump the value subtracted) and recording the value of the counters left underneath the number line.



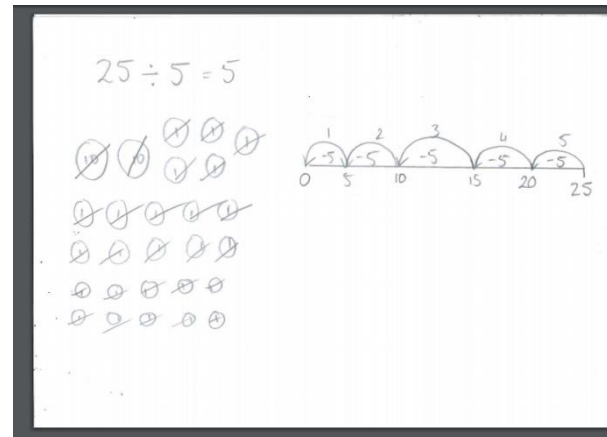
3) Repeat step 2 until there are no counters left (you may need to exchange 10s counters for 1s counters in order to complete this method), recording each jump on the number line. If there are not enough counters left to continue subtracting the smallest number, these can be recorded as a remainder.



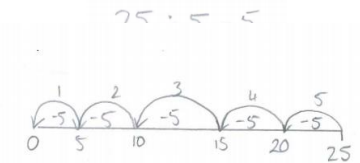
4) Count the number of jumps made on the number line.



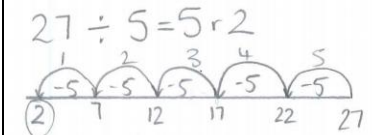
Children can follow the procedure shown in the abstract, using images of the place value counters alongside the number line.



Children will use a number line to support division without the use of place value counters (physical or drawn).



A remainder by looking at the last number landed upon when no further jumps can be made.



**Short Division
(compact/'bus
stop' method)
with and
without
remainders.**

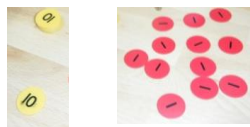
Children to work with base 10 or place value counters to support division of two- or more digit numbers by a 1-digit number.

$$72 \div 6 =$$

Make the largest number with place value counters. (or base 10).



Starting with the 10s counters, stack the counters into piles of the number you are dividing by. Any that can't be stacked need to be exchanged for 1s counters.



Now stack the ones counters in to piles of the number you are dividing by.

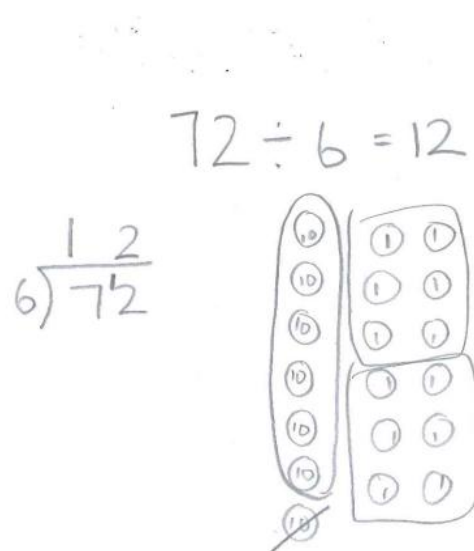


The answer is given by counting the number of piles you have in each column. The number of 10s piles gives you your 10s digit, the number of 1s piles gives you your 1s digit. E.g. 12



Any unused ones counters are recorded as a remainder.

Children can follow the same steps as in the abstract, using drawings of the counters to support.

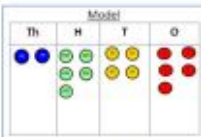


Children use their times table knowledge to use short division as a written method.

$$72 \div 6 = 12$$

$$\begin{array}{r} 12 \\ 6 \overline{) 72} \end{array}$$

Long division



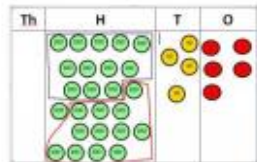
$2544 \div 12$
How many groups of 12 thousands do we have?
None

Exchange 2 thousand for 20 hundreds.



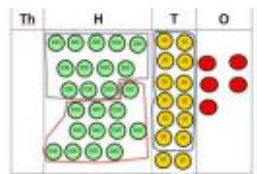
$$12 \overline{) 2544}$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



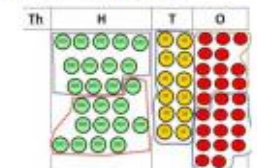
$$12 \overline{) 2544} \\ \underline{24} \\ 1$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0$$

Instead of using physical counters, students can draw the counters and circle the groups.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

$$\begin{array}{r} 0318r5 \\ 20 \overline{) 6365} \\ \underline{60} \\ 36 \\ \underline{20} \\ 165 \\ \underline{160} \\ 5 \end{array}$$